

Discovering Math: Exploring Geometry

Teacher's Guide

Grade Level: 6–8

Curriculum Focus: Mathematics

Lesson Duration: Three class periods

Program Description

Discovering Math: Exploring Geometry – From methods of geometric construction and three-dimensional figures to representations and transformations, introduce students to more advanced properties and concepts of geometry.

Lesson Plan

Student Objectives

- Create three-dimensional figures.
- Construct a three-dimensional model of a city using similar and congruent figures and geometric transformations.
- Create a two-dimensional representation of their city.
- Find the surface area of three-dimensional figures.

Materials

- *Discovering Math: Exploring Geometry* video
- Building Construction Chart (see below)
- Surface Area Chart (see below)
- Models of three-dimensional figures
- Modeling clay
- Paper
- Tape
- Scissors
- Measuring tools
- Compass
- Cardboard to use as base for city (size to be determined by teacher)

Procedures

1. Tell students that they will be constructing three-dimensional models of a city. Discuss the directions and parameters of the project.
 - Construct a three-dimensional model of a city that has at least 10 buildings. The buildings must include two similar figures, two congruent figures, and two geometric transformations. Each building must be labeled and the figures used identified in the Building Construction Chart.
 - Use paper or modeling clay to create the three-dimensional figures that will be used to construct the buildings.
 - Add streets to the city using straight edges.
 - Students should be able to describe the streets using the terms parallel, perpendicular, and intersecting.
2. Ask students to describe surface area. Display a three-dimensional figure and model how to calculate its surface area. Remind students to use the Pythagorean theorem when find the length of sides of right triangles.
 - Ask students trade cities with a classmate.
 - Have students calculate the surface area of at least five buildings in their classmate's city. They should use an appropriate measuring tool and record the surface area of each building on the Surface Area Chart.
 - Have students identify parallel, perpendicular, and intersecting streets in the city.
3. Ask students to identify the three types of two-dimensional representations presented in the video. Discuss the properties of planar cross sections, scale drawings, and blueprints. Have students create a two-dimensional representation of their city.
 - Remind students that two-dimensional representations should be drawn to scale. If needed, drawing to scale should be modeled and explained.
 - Have students share their representations with the class. They should compare their two- and three-dimensional representations of the city and discuss the advantages and disadvantages of each type of representations.

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students produced a complete city following all of the parameters; clearly demonstrated the ability to create identify three-dimensional figures; clearly demonstrated the ability to perform geometric transformations; clearly demonstrated the ability to create and identify similar and congruent figures; clearly demonstrated the ability to calculate surface area of three-dimensional figures; and clearly demonstrated the ability to create a two-dimensional representation of a three-dimensional object.

- **2 points:** Students produced an adequate city including at least eight buildings; demonstrated the ability to create and identify three-dimensional figures 80% of the time; demonstrated the ability to perform at least one geometric transformation; demonstrated the ability to create and identify at least one similar or congruent figure; demonstrated the ability to calculate surface area of three to four three-dimensional figures; and satisfactorily demonstrated the ability to create a two-dimensional representation of a three-dimensional object.
- **1 point:** Students created an incomplete city with less than eight buildings; demonstrated the ability to create and identify three-dimensional figures less than 80% of the time; did not demonstrate the ability to perform geometric transformations; did not demonstrate the ability to create and identify similar or congruent figures; demonstrated the ability to calculate surface area of less than three three-dimensional figures; and did not demonstrate the ability to create a two-dimensional representation of a three-dimensional object.

Vocabulary

blueprint

Definition: Two-dimensional representation of a three-dimensional object that shows the relationships between all the parts of an object from one viewpoint

Context: The architect drew a blueprint to represent the plans for the new house.

congruent figure

Definition: A figure that has the exact shape and size of another figure with a ratio of 1:1 for the corresponding parts

Context: Sue and Mary both drew squares with three-inch-long sides on the board, so they drew congruent figures.

dilation

Definition: A geometric transformation that changes a figure's size, but its shape, orientation, and location stay the same

Context: The animator performed a geometric dilation in order to create the illusion that the figure was getting larger without changing its shape, orientation, or location.

planar cross section

Definition: A two-dimensional representation that shows the inside of a three-dimensional object to illustrate the spatial relationships within the object

Context: Seismologists use planar cross sections to show the layers of the Earth.

rotation

Definition: A geometric transformation that changes a figure's orientation and location, but not its size or shape, by rotating it around an axis

Context: The animator performed a geometric rotation to make the figure spin.

scale drawing

Definition: A two-dimensional representation of an object using a scale that is the ratio of the size of the drawing to the actual size of the object

Context: A map is a scale drawing.

similar figures

Definition: Figures that have the same shape but not necessarily the same size, congruent corresponding angles, and equal ratios for the lengths of corresponding sides

Context: Triangles ABC and DEF are similar figures.

surface area

Definition: The sum of the areas of the outside surfaces of a three-dimensional figure

Context: The surface area of a cube with sides that measure 5 feet is 150 square feet.

translation

Definition: A geometric transformation that changes a figure's location, but its size and shape remain the same

Context: The animator performed a geometric translation on the figure to create the illusion of movement.

Academic Standards

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education addresses 14 content areas. To view the standards and benchmarks, visit

<http://www.mcrel.org/compendium/browse.asp>.

This lesson plan addresses the following benchmarks:

- Uses geometric methods (i.e., an unmarked straightedge and a compass using an algorithm) to complete basic geometric constructions (e.g., perpendicular bisector of a line segment, angle bisector).
- Understands the defining properties of three-dimensional figures (e.g., a cube has edges with equal lengths, faces with equal areas and congruent shapes, right angle corners).
- Understands the defining properties of triangles (e.g., the sum of the measures of two sides of a triangle must be greater than the measure of the third side).
- Understands geometric transformations of figures (e.g., rotations, translations, dilations).
- Understands the relationships between two- and three-dimensional representations of a figure (e.g., scale drawings, blueprints, planar cross sections).
- Understands the mathematical concepts of similarity (e.g., scale, proportion, growth rates) and congruency.
- Understands the basic concept of the Pythagorean theorem.

National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics (NCTM) has developed national standards to provide guidelines for teaching mathematics. To view the standards online, go to <http://standards.nctm.org>.

This lesson plan addresses the following standards:

- Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
 - Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.
 - Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
 - Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.
 - Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.
 - Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.
 - Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
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Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the [Discoveryschool.com](http://www.discoveryschool.com) Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- <http://school.discovery.com/teachingtools/teachingtools.html>
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DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How to Use the DVD

The DVD starting screen has the following options:

Play Video – This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index – Here the video is divided into chapters indicated by title. Each chapter is then divided into four sections indicated by video thumbnail icons; brief descriptions are noted for each

section. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Quiz – Each chapter has four interactive quiz questions correlated to each of the chapter's four sections.

Standards Link – Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources – This screen gives the technical support number and Web site address.

Video Index

I. Basic Geometric Constructions (6 min.)

Basic Geometric Constructions: Introduction

Architects and engineers create mechanical drawings to represent future structures. See why accurate lines, angles, and shapes are crucial.

Example 1: Copying a Triangle

Learn how to create a congruent triangle using a straight edge and compass.

Example 2: Constructing an Angle Bisector

An angle bisector is a ray that splits an angle into two congruent angles. Learn how to create an angle bisector using a compass and straight edge.

Example 3: Constructing a Perpendicular Bisector

A perpendicular bisector intersects a line segment, creating four 90-degree angles. Learn how to create a perpendicular bisector using a compass and straight edge.

II. Defining Properties of 3-D Figures (8 min.)

Defining Properties of 3-D Figures: Introduction

A three-dimensional figure has points outside a single plane. Explore the properties of a polyhedron, a 3-D object with polygons as faces.

Example 1: Properties of Cubes and Spheres

Explore the properties of a cube, a prism with square faces, right angle corners, equal edges, and congruent areas on each face, and a sphere, a three-dimensional object with infinite congruent radii.

Example 2: Properties of Rectangular Prisms and Cylinders

Investigate the properties of rectangular prisms, three-dimensional figures with congruent rectangular bases and congruent opposite parallelogram faces, and cylinders, three-dimensional objects with congruent parallel circular bases with a connecting surface.

Example 3: Properties of Pyramids and Cones

Explore the properties of pyramids, polygonal bases with triangular faces, and cones, circular bases with a surface created by points on the circle connecting to a non-coplanar point.

III. Defining Properties of Triangles (9 min.)

Defining Properties of Triangles: Introduction

Explore the properties of triangles and see how they can combine to form other shapes.

Example 1: Properties of Triangles

Triangles have three sides and three angles. The sum of the angles equals 180 degrees, and the sum of the lengths of two sides is always greater than the length of the third side.

Example 2: Angles of a Triangle

The sum of the angles in a triangle equals 180 degrees. Find out how to calculate an unknown angle measurement in a triangle.

Example 3: Special Triangles

Explore the properties of unique triangles: Equilateral triangles have equal sides and angles, isosceles triangles have two equal angles and sides, and scalene triangles have three different-size sides and angles.

IV. Geometric Transformations (8 min.)

Geometric Transformations: Introduction

Witness the creation of a fictional planet using geometric shapes and transformations. Use geometric translations, rotations, and dilations to manipulate a shape's size, orientation, or location.

Example 1: Translations

Geometric translations change a figure's location, but its size and shape remain the same. See how translations create movement that can be measured.

Example 2: Rotations

Geometric rotations change a figure's orientation and location, but its size and shape remain the same. See how rotations allow a figure to rotate around an axis on a circle.

Example 3: Dilations

Geometric dilations change a figure's size, but its shape, orientation, and location stay the same. Use a scaling factor to dilate a figure so the new shape is proportional to the original shape.

V. Cross-Dimensional Representations (8 min.)

Cross-Dimensional Representations: Introduction

See how two-dimensional drawings are useful representations of three-dimensional objects. Examples of two-dimensional drawings are scaled drawings, blue prints, architectural plans, and cross sectional drawings.

Example 1: Scale Drawings

A map is an example of a scale drawing, showing one view of a three-dimensional area. Learn how map scales indicate the ratio of a distance on a map to a distance in the real world.

Example 2: Blueprints

See how architects use blueprints to design a building. Blueprints are two-dimensional models that show the relationships between all parts of the object from one viewpoint at a time.

Example 3: Planar Cross Sections

Watch seismologists use planar cross sections to study Earth's interior. Planar cross sections are two-dimensional representations that illustrate spatial relationships within an object.

VI. Similarity and Congruency (9 min.)

Similarity and Congruency: Introduction

Congruent figures have exactly the same shape and size, and the ratio of their corresponding parts is 1:1. Similar figures have the same shape but different sizes, and their corresponding parts are proportional.

Example 1: Proportion and Scale

A model of the United States Capitol represents similar figures. Learn how ratio determines the scale of one object in relation to another.

Example 2: Congruence and Similarity

Similar objects have the same shape and their corresponding parts have the same ratio. Investigate how all flags are similar.

Example 3: Proportionate Growth

Ratios show the relationship between measurements. Use ratios to describe the similarities between parent and young sharks and see how they grow proportionally.

VII. Pythagorean Theorem (8 min.)

Pythagorean Theorem: Introduction

Explore the Pythagorean theorem and see how the hypotenuse is the side opposite the right angle in a right triangle.

Example 1: An Informal Proof

Use the Pythagorean theorem to calculate one side of a right triangle when the measures of the other two sides are known, and see how geometric models explain the theorem.

Example 2: Find the Hypotenuse

Use the Pythagorean theorem to determine the shortest distance between two locations in Washington, D.C.

Example 3: Find the Base

Use the Pythagorean theorem to determine the measurements needed to create a new sail for a sailboat.

Quiz

I. Basic Geometric Constructions

1. Hannah has a picture of triangle ABC. Identify the two tools she can use to construct a triangle that is congruent to triangle ABC.
 - A. ruler and tape measure
 - B. compass and bisector
 - C. compass and straight edge
 - D. protractor and tape measure

Answer: C

2. Lori constructed a ray that divided angle CGF into two congruent angles. What did she construct?
 - A. an angle divider
 - B. a bisecting angle
 - C. an angle bisector
 - D. a perpendicular bisector

Answer: C

3. What types of angles are formed by a perpendicular bisector?
 - A. acute
 - B. right
 - C. obtuse
 - D. bisecting

Answer: B

II. Defining Properties of 3-D Figures

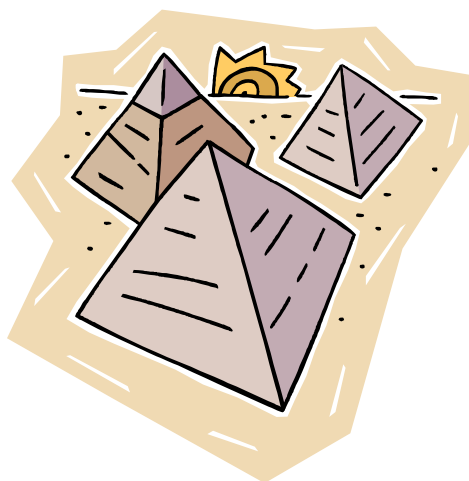
1. Identify the polyhedron.
 - A. circle
 - B. square
 - C. triangle
 - D. pyramid
2. What is the surface area of a cube with sides that each measure seven feet?
 - A. 49 square feet
 - B. 84 square feet
 - C. 252 square feet
 - D. 294 square feet

Answer: D

3. Charles sees a three-dimensional figure with congruent parallel circular bases with a connecting surface. What is it?
- A. a cone
 - B. a sphere
 - C. a cylinder
 - D. a rectangular prism

Answer: C

4. The figures below have polygonal bases and triangular faces. What are they?
- A. cones
 - B. triangles
 - C. pyramids
 - D. cylinders



Answer: C

III. Defining Properties of Triangles

1. Tina measures sides DE and DF of triangle DEF. Side DE is 5 inches long and side DF is 12 inches long. What is the length of side EF?
- A. 21 inches
 - B. 19 inches
 - C. 17 inches
 - D. 13 inches

Answer: D

2. Lara measures angles 1 and 2 of triangle ABC. Angle 1 measures 35 degrees and angle 2 measures 100 degrees. What is the measure of angle 3?
- A. 35 degrees
 - B. 45 degrees
 - C. 55 degrees
 - D. 80 degrees

Answer: B

3. Ken created a triangle with angles measuring 30 degrees, 30 degrees, and 120 degrees. What type of triangle did he create?
- A. scalene
 - B. isosceles
 - C. equilateral
 - D. equiangular

Answer: B

IV. Geometric Transformations

1. Identify the geometric transformation that changes a figure's location but not its size and shape.
- A. dilation
 - B. rotation
 - C. reflection
 - D. translation

Answer: D

2. An animator moved a figure around the axis of a circle. What type of geometric transformation was performed?
- A. rotation
 - B. reflection
 - C. translation
 - D. scaling factor

Answer: A

3. Identify the correct description of a geometric dilation.
- A. A scaling factor is used to change the size of a figure.
 - B. A figure is moved along a line to change its location and shape.
 - C. A figure is rotated around the axis of a circle to change its size.
 - D. A scaling factor is used to make a figure's location and shape change.

Answer: A

V. Cross-Dimensional Representations

1. Point A is 3 inches from point B on the map. The map scale shows that 1 inch equals 42 miles. How far is point A from point B?
 - A. 126 miles
 - B. 126.5 miles
 - C. 147 miles
 - D. 152 miles

Answer: C

2. Students creating a representation of a building must show the relationships between the parts of the building. What type of two-dimensional representation should they use?
 - A. map
 - B. blueprint
 - C. three-dimensional model
 - D. planar cross sectional drawing

Answer: B

3. Allan must represent the layers of the Earth. What is the best way to represent this information in two dimensions?
 - A. Draw a map of the Earth.
 - B. Create a blueprint of the Earth.
 - C. Draw a planar cross section of the Earth.
 - D. Create a model of the Earth showing the layers.

Answer: C

VI. Similarity and Congruency

1. Olivia drew two congruent shapes. Which statement is true?
 - A. The shapes are the same size and shape.
 - B. The shapes have the same shape but are different sizes.
 - C. The shapes have the same size but are different shapes.
 - D. The corresponding parts of the shapes have a ratio of 1:4.

Answer: A

2. Victor created two similar shapes and the ratio of the corresponding sides is 1:3. If side A of the first shape is four feet, what is the length of side A on the second shape?
- A. 1 foot
 - B. 7 feet
 - C. 12 feet
 - D. 16 feet

Answer: C

3. Which statement is true?
- A. All congruent shapes are similar.
 - B. All similar shapes are congruent.
 - C. The ratio of corresponding sides in similar figures is always 1:1.
 - D. The ratio of corresponding sides in congruent shapes is always 1:2.

Answer: A

4. The ratio of a shark's fin to its body is 1:4. If the fin is 1 foot long, how long is the body?
- A. 4 feet
 - B. 5 feet
 - C. 5 feet
 - D. 6 feet

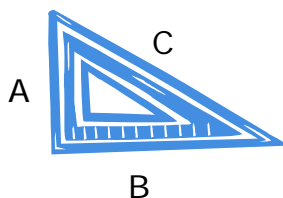
Answer: D

VII. Pythagorean Theorem

1. The base of right triangle BHJ is three feet and the height is four feet. Use the Pythagorean theorem to identify the length of the hypotenuse.
- A. 2 feet
 - B. 5 feet
 - C. 7 feet
 - D. 25 feet

Answer: B

2. In right triangle ABC, A measures 6 inches and C measures 10 inches. What is the length of B?
- A. 64 inches
 - B. 36 inches
 - C. 12 inches
 - D. 8 inches



Answer: D

3. In a right triangle the hypotenuse measures 20 meters, and the base measures 16 meters. What is the height?
- A. 8 meters
 - B. 10 meters
 - C. 12 meters
 - D. 28 meters

Answer: C

Building Construction Chart

Identify the buildings and the figures used in their construction in the chart below.

Building	Figures Used

Surface Area Chart

Find the surface area of five buildings in your classmate's city.

Building	Surface Area